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HYDRODYNAMIC INSTABILITY AND REGIMES OF FRAGMENTATION OF DROPS

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Estimates of the dispersal parameters are obtained and an explanation is proposed for the mechanism of various types of destruction on the basis of a linear analysis of the stability of the surface of a drop.

The fragmentation of liquid drops and jets by a high-speed gas stream is an important process in many industrial installations and can exert considerable influence on the flow of gas–drop mixtures. Because of the complexity of the physical phenomena comprising this process, fragmentation is studied predominantly by empirical methods, so that it has been well investigated experimentally but a complete theoretical model does not yet exist [1, 2], preventing one from obtaining reliable estimates of the sizes of the droplets torn off and the time of their separation and clarifying the various types of destruction.

In [3] an attempt was made to give a unified explanation of fragmentation as the manifestation of hydrodynamic instability of the drop surface. In that paper a mathematical model of a fragmenting drop was constructed on the basis of a solution of the problem of the stability of an accelerated tangential velocity discontinuity, and it was concluded that the description of the phenomenon is adequate, despite the definite quantitative inconsistency.

Further refinement of the model, connected with an investigation of the inviscid instability of the interface between two media with the property of continuity of the velocity profile inherent to actual fluxes, showed [4] that the model of a tangential discontinuity can only serve as a rough approximation, since the decrease in the velocities of the media in the boundary layers has a considerable stabilizing action.

For two-phase systems of the air–water, air–kerosene, etc. type the instability of the continuous profile is due to gradient flow of the denser liquid in the boundary layer and is described by the dispersion relation for the dimensionless "frequency" $z = \omega\delta/V_i$ of a disturbance of the type $\exp(ihx - i\omega t)$,

$$(z-\Delta)[(z-\Delta)(z+\Delta A)+\Delta(1-A)]=(z-\Delta A)\left[\alpha\Delta^3\,\mathrm{We}_b^{-1}-\frac{\Delta\delta g\cos\varphi}{V_i^2}\right], \tag{1}$$

where $\Delta = h\delta$; $A = (1 - \exp(-2\Delta))/2$.

An analysis of the development of gradient instability under the conditions of the flow of a gas stream over a drop is of interest. Below we find the conditions for the appearance of instability, estimates of the main characteristics of the destruction are obtained, and certain conclusions about the character of the destruction are drawn on this basis.

First of all we must investigate the vicinity of the rim of the drop ($\varphi=\pi/2$), where the separation of particles is observed experimentally. Here the influence of acceleration is

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small, $g\cos\varphi \simeq 0$, and an analysis of Eq. (1) shows the existence of a root possessing growing perturbations and having a periodic character. For it in Fig. 1 we plot graphs of the dimensionless wave number Δ^* and the increment $\mathrm{Im}(z(\Delta^*))$ of growth of the amplitude of the wave growing at the maximum rate. The quantities Δ^* and $z(\Delta^*)$, and hence the development of unstable perturbations, depend on the values of the Weber number We_b of flow in the liquid boundary layer. As is seen, an unstable root exists for $We_b \gtrsim 0.004$, disappearing for $We_b < 0.004$ due to stabilization of disturbances by surface tension, while for $We_b \gtrsim 0.025$ the forces of surface tension are small compared with the hydrodynamic forces, resulting in the root being independent of We_b. In this case Eq. (1) is simplified owing to the isolation of the real root $z = \Delta$ and it yields the constant values $\Delta^* \simeq 1.22$ and $\mathrm{Im}(z(\Delta^*)) \simeq 0.24$ characteristic for gradient instability.

We shall treat fragmentation as a local manifestation of instability of the surface of a drop, for which we divide it into a system of elementary plane areas characterized by the angle φ. For a given law $V_g(\varphi)$ of potential flow over a deformed drop and a given distribution $\delta(\varphi) = l(\varphi) d_0 \mathrm{Re}^{-0.5}$ of thickness of the boundary layer at it we write the expression for the variation of We_b along the surface:

$$\mathrm{We}_b(\varphi) = \alpha l(\varphi) Q(\varphi) \mathrm{We}\,\mathrm{Re}^{-0.5},\quad \alpha = \frac{\rho_\infty}{\rho_l},\quad Q = \frac{\rho_g V_g^2}{\rho_\infty V_\infty^2}. \tag{2}$$

From (2) it follows that the stability of the surface of a drop is determined both by the regime of flow over it (the distribution of lQ) and by the general properties of the stream (the values of $\mathrm{WeRe}^{-0.5}$), and the second factor, varying in a wide range, can considerably alter the regime of destruction of the drop as a whole. In this connection the parameter $\mathrm{WeRe}^{-0.5}$ plays an important role in determining the character of the fragmentation. In fact, a large number of experiments on the character of fragmentation. In fact, a large number of experiments on the character of fragmentation can be systematized [2] with the use of just this parameter; in Fig. 2 we show the regions I, II, and III of different regimes of fragmentation from this paper.

Let us estimate the critical conditions for the appearance of instability. The condition $We_b \gtrsim 0.004$ for $\alpha = 0.0013$, $\varphi = \pi/2$, $l = 5$, and $Q = 2$ with allowance for (2) is equivalent to

$$(\mathrm{We}\,\mathrm{Re}^{-0.5})_{cr} \gtrsim 0.3. \tag{3}$$

This condition for the appearance of periodic unstable perturbations on the surface of a drop agrees well with empirical criteria for the existence of fragmentation with separation of the surface layer [2, 5]. It is obvious that only those unstable perturbations for which the half-wavelength is less than the thickness of the deformed drop can lead to dispersal. We implement this condition for different values of We and Re by using the relations $\Delta^*(We_b)$ and (2), and we obtain a relation isolating the region A of possible periodic disposal in the plane We–$\mathrm{WeRe}^{-0.5}$ of fragmentation regimes. It is bounded on the left and below by the curve B (Fig. 2) and essentially coincides with the empirically constructed regions II and III of the existence of fragmentation with separation of the surface layer. This coincidence allows us to explain the destruction in regions II and III as periodic dispersal due to gradient instability of the drop surface.

The size of the particles torn off from each elementary area can be estimated as the length $\lambda^*(\varphi)$ of the unstable wave developing on it, while the separation time can be estimated as the time of the e-fold growth of its amplitude: $\tau_e^*(\varphi) = \alpha^{0.5} V_\infty d_0^{-1} \mathrm{Im}^{-1}(\omega(\Delta^*))$. In the simplest approximation we assume that the potential flow over the deformed drop corresponds to potential flow over a sphere, i.e., $V_g(\varphi) = 1.5 V_\infty \sin\varphi$, while for the thickness of the boundary layer at the drop we use Eq. (7) obtained in [6]:

$$l(\varphi) = 1.1\,[(6\varphi - 4\sin 2\varphi + 0.5\sin 4\varphi)/\sin^5\varphi]^{0.5}.$$

The results of a calculation of $\lambda^*(\varphi)$ and $\tau_e^*(\varphi)$ (see Table 1) made using the relations $\Delta^* \times (We_b)$, $\mathrm{Im}(z^*(We_b))$, and (2) allow one to obtain quantitative estimates of the dispersal parameters for each elementary area characterized by the angle φ as a function of the values of the determining parameters We and Re, as well as to draw conclusions of a qualitative character.

For values of We and Re corresponding to region A there exists a critical point on the surface of the drop at which the local value of $We_b(\varphi_{cr})$ equals the critical value: $We_b \simeq 0.004$. It divides the surface of the drop into two zones: one stable against periodic

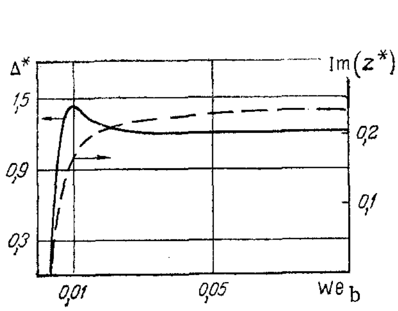

Fig. 1

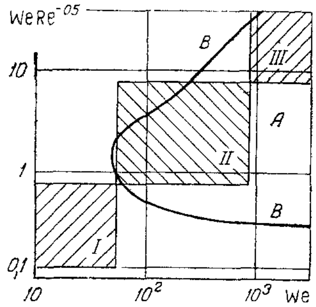

Fig. 2

Fig. 1. Graphs of the functions $\Delta^*(We_b)$ and $Im(z^*(We_b))$.

Fig. 2. Empirical regions of fragmentation regimes I, II, and III [2] and region A of destruction in the regime of periodic dispersal.

perturbations ($\varphi<\varphi_{cr}$) and an unstable zone located near the rim, $\varphi_{cr} < \varphi \leqslant \pi/2$. With an increase in $WeRe^{-0.5}$ the critical points shifts away from the rim toward the front stagnation point, which leads to an increase in the area from which dispersal occurs. Thus, an explanation is found for the experimental fact [7] of particle separation far ahead of the equator of a drop, at $\varphi < \pi/2$, which cannot be explained within the framework of any theory proposed up to now, such as the separation of the boundary layer. This also explains the good agreement between experiment and the results of a calculation of the transverse deformation of a drop made without satisfying the continuity equation [7]. The calculated data were compared with the size of the region containing a large number of already separated particles and therefore not having the property of continuity (Fig. 3c).

The dispersal period over a large part of the surface levels out with an increase in $WeRe^{-0.5}$. Together with the increase in the dispersal area this leads to an increase in the rate of mass loss and creates the impression of a sudden intense fragmentation, characteristic for region III and known as "explosive decay."

The condition when surface tension does not affect the development of perturbations, $We_b \gtrsim 0.025$, is equivalent to the following: $WeRe^{-0.5} \gtrsim 2$. In this region the fragmentation parameters are expressed through the Reynolds number:

$$\frac{\lambda^*}{d_0} \simeq 5\frac{l}{Re^{0.5}}, \quad \tau_e^* \simeq 2.7\frac{l}{Re^{0.5}}.$$

Since the volume carried off by particles from an elementary area is proportional to $(\lambda^*)^2$ (with allowance for axial symmetry) while the number of particles is proportional to $(\lambda^*)^{-1}$, the rate of mass loss proves to be approximately constant (in agreement with experiment [7]) and equal to $dM/d\tau \simeq -0.3$. Hence the time of complete destruction is $\tau_d \simeq 3.3$, which agrees with experiment [7]: $\tau_d \simeq 3.5$.

TABLE 1. Calculated Values of $\lambda^* d_0^{-1} Re^{0.5}$ (numerator) and $\tau_e^* Re^{0.5}$ (denominator) (STA: stable sections of drop surface)

$WeRe^{-0,5}$	sin φ										
	1,00	0,95	0,90	0,85	0,75	0,65	0,60	0,50	0,35	0,30	0,20
30,0	25,6/13,8	15,8/9,1	14,1/8,5	12,9/8,3	11,4/8,3	10,4/8,9	9,9/9,3	9,2/10,9	8,1/16,3	7,5/21,0	16,1/99,7
10,0	25,6/13,9	15,8/9,2	14,0/8,7	12,8/8,5	11,1/8,9	10,0/9,8	9,5/10,3	8,44/13,5	14,0/48,6	58,6/245,0	STA
3,0	25,1/14,6	15,2/10,1	13,0/9,7	11,8/10,4	9,8/11,7	10,4/17,6	14,0/27,9	257,0/345,3	STA	STA	STA
2,0	24,7/15,4	14,5/11,4	12,4/11,5	11,1/11,7	11,8/17,5	28,5/55,0	136,8/286,4	STA	STA	STA	STA
0,8	22,2/19,2	20,7/25,5	35,3/49,1	117,7/174,0	STA	STA	STA	STA	STA	STA	STA

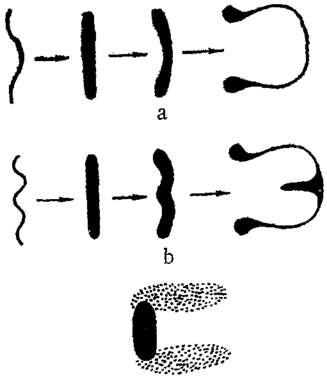

Fig. 3

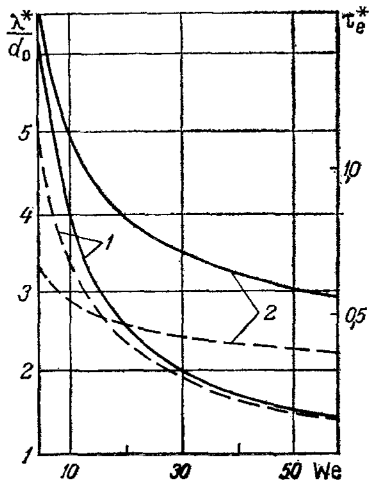

Fig. 4

Fig. 3. Schemes of destruction of a drop: a) "parachute" type; b) "claviform" type; c) in the dispersal regime.

Fig. 4. Graphs of the functions $\lambda^*/d_0(\mathrm{We})$ (1) and $\tau_e^*(\mathrm{We})$ (2) for Taylor-unstable perturbations in the region of low values of We. Dashed curves: classic solution for half-spaces; solid lines: solution for a sheet.

On that part of the surface of a drop where one cannot take $g\cos\varphi = 0$ one must take into account the Rayleigh—Taylor instability factor. The analysis of Eq. (1) and its application are difficult in this case and they can be done numerically for concrete values of the parameters within the framework of the model of [3].

In a number of experimental and theoretical papers [8] it has been established that aperiodic unstable Taylor perturbations in the nonlinear stage of development have a constant rate of growth of amplitude, $U = 0.338(\lambda^* g)^{0.5} \approx \mathrm{We}^{-0.25}$. Hence it follows that in region III the time of "broaching" of a drop by aperiodic perturbations should grow as $\mathrm{We}^{0.25}$, which is at variance with numerous experiments [7], $\tau_d \sim \mathrm{We}^{-0.25}$, and indicates the insignificant role of Taylor instability in region III. Evidently, this type of instability can be realized only in the concluding stage and destroy the part of the drop remaining after the action of peridic instability.

In regions I and II the role of Taylor instability is important. We can show that the action of just this type of instability can explain destruction of the "parachute" and "claviform" types. The stabilization of gradient instability reduces our problem to the investigation of the stability of an accelerating layer of liquid. Such a problem was analyzed in [3], where a dispersion relation and a transcendental equation for the length of the fastest growing wave were obtained. In Fig. 4 we present the solution of these equations in the region of small values of We with allowance for the axial symmetry of the problem for the values $k = 2$, $d = 3d_0$, and $V_g = 0$ in the form of $\lambda^*(\mathrm{We})$ and $\tau_e^*(\mathrm{We})$ graphs. Here the conditions for a free surface were adopted because of the strong deformation of the drop and its poor streamlining on the back side.

The results of the calculation show that in the range of $5 \lesssim \mathrm{We} \lesssim 60$ the drop is subject to the action of perturbations with a wavelength greater than the initial diameter and comparable with the transverse size d of the deformed drop. When the condition $\lambda^*/2 \lesssim d$ is satisfied, in particular, the unstable perturbation can lead to destruction. This condition is satisfied for $\mathrm{We} > \mathrm{We}_{cr} \simeq 5$. The value of We_{cr} coincides with the empirically determined values of the critical Weber number.

The action of a perturbation with $\lambda^*/2 \simeq d$ initially leads to warping of the flattened drop (Fig. 3a) and then to the continuous, by virtue of the aperiodicity of the instability, elongation of the resulting cavity, which in the nonlinear stage expands in the direction transverse to the acceleration, which is characteristic for a Taylor instability [9], and forms a "parachute."

With an increase in We, λ^* decreases and a situation arises when three half-waves of the perturbation fit into the transverse diameter of the disk: $3\lambda^*/2 \simeq d$. In this case the gas cavities also expand and form "sacks," while the part of the perturbation facing the gas forms a peak (this is also a characteristic property of Taylor-unstable perturbations [9]) and the pestle peculiar to the "claviform" develops (Fig. 3b). Curve 1 of Fig. 4 shows that these conditions are satisfied for $We > We_{cr} \simeq 30$.

For We > 60 several wavelengths of perturbation act on the liquid disk at once. Moreover, the appearance of periodically unstable perturbations on the rim of the drop is possible under these conditions. Evidently, the combined action of these two kinds of unstable perturbations leads to the chaotic type of destruction. The above comparisons show the good qualitative and quantitative agreement of the proposed model with experiment. The unified description of all the main types of destruction from the standpoint of the theory of hydrodynamic stability, permitting the construction of a simple mathematical model of a fragmenting drop on this basis, must be included among the advantages of the model.

It must be noted that such an approach can be used to describe the interaction between phases leading to destruction in other systems such as bubbles and films.

NOTATION

$h = 2\pi/\lambda$, wave number of perturbation; δ, thickness of the boundary layer in the liquid; V_∞, ρ_∞, velocity and density of the gas stream impinging on the drop; V_g, ρ_g, values of the gas velocity and density on an elementary area; V_i, velocity at the interface between media; ρ_l, liquid density; σ, surface tension coefficient; d_0, initial drop diameter; d, transverse deformation of the drop; $g = \alpha k V_\infty^2 d_0^{-1}$, acceleration of the drop; $We = \rho_\infty V_\infty^2 d_0 \sigma^{-1}$, Weber number of the gas stream; $Re = \rho_\infty V_\infty d_0 \mu_g^{-1}$, Reynolds number of the gas stream; $We_b = \rho_g V_i^2 \delta \sigma^{-1}$, Weber number of flow in the liquid boundary layer; φ, angle between the velocity vector of the impinging stream and the normal to the drop surface.

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